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Strong decays $B_{s0} \to B_s\pi$ and $B_{s1} \to B_s^*\pi$ with light-cone QCD sum rules

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Abstract. In this article, we calculate the strong coupling constants $g_{B_{s0}B_s\eta}$ and $g_{B_{s1}B_s^*\eta}$ with the light-cone QCD sum rules. Then we take into account the small η–π^0 transition matrix according to Dashen's theorem, and we obtain the small decay widths for the isospin violation processes $B_{s0} \to B_s\eta \to B_s\pi^0$ and $B_{s1} \to B_s^*\eta \to B_s^*\pi^0$. We can search the strange-bottomed $(0^+, 1^+)$ mesons B_{s0} and B_{s1} in the invariant $B_s\pi^0$ and $B_s^*\pi^0$ mass distributions, respectively.

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1 Introduction

Recently, the CDF Collaboration reported the first observation of two narrow resonances consistent with the orbitally excited P-wave B_s mesons using $1\,\mathrm{fb}^{-1}$ of $p\bar{p}$ collisions at $\sqrt{s} = 1.96\,\mathrm{TeV}$ collected with the CDF II detector at the Fermilab Tevatron [1]. The masses of the two states are $M(B_{s1}^*) = (5829.4 \pm 0.7)\,\mathrm{MeV}$ and $M(B_{s2}^*) = (5839.7 \pm 0.7)\,\mathrm{MeV}$, and they can be assigned as the $J^P = (1^+, 2^+)$ states in the heavy quark effective theory [2]. The D0 Collaboration reports the direct observation of the excited P-wave state B_{s2}^* in fully reconstructed decays to B^+K^-. The mass of the B_{s2}^* meson is measured to be $(5839.6 \pm 1.1 \pm 0.7)\,\mathrm{MeV}$ [3]. The B_s states with spin-parity $J^P = (0^+, 1^+)$ still lack experimental evidence.

The masses of the B_s mesons with $(0^+, 1^+)$ have been estimated with the potential quark models, heavy quark effective theory and lattice QCD [4–16]; the values are different from each other. In [17], we study the masses of the strange-bottomed $(0^+, 1^+)$ mesons with the QCD sum rules and observe that the central values are below the corresponding BK and B^*K thresholds, respectively. The decays $B_{s0} \to BK$ and $B_{s1} \to B^*K$ are kinematically forbidden. In previous works, the mesons $f_0(980)$, $a_0(980)$, D_{s0}, D_{s1}, B_{s0} and B_{s1} were taken as the conventional $q\bar{q}$, $c\bar{s}$ and $b\bar{s}$ states, respectively, and the values of the strong coupling constants g_{f_0KK}, g_{a_0KK}, $g_{D_{s0}DK}$, $g_{D_{s1}D^*K}$, $g_{B_{s0}BK}$ and $g_{B_{s1}B^*K}$ are calculated with the light-cone QCD sum rules [18–23]. The large values of the strong coupling constants support the hadronic dressing mechanism [24–26]. Those mesons may have small $q\bar{q}$, $c\bar{s}$ and $b\bar{s}$ kernels of the typical $q\bar{q}$, $c\bar{s}$ and $b\bar{s}$ mesons size, respectively; strong couplings to the virtual intermediate hadronic states (or the virtual mesons loops) may result in smaller masses than the conventional $q\bar{q}$, $c\bar{s}$ and $b\bar{s}$ mesons in the potential quark models and enrich the pure $q\bar{q}$, $c\bar{s}$ and $b\bar{s}$ states with other components [14, 18–23, 27–29].

The P-wave heavy mesons B_{s0} and B_{s1} can decay through the isospin violation processes $B_{s0} \to B_s\eta \to B_s\pi^0$ and $B_{s1} \to B_s^*\eta \to B_s^*\pi^0$, respectively. The η–π^0 transition matrix is very small according to Dashen's theorem [30], $t_{\eta\pi} = \langle\pi^0|\mathcal{H}|\eta\rangle = -0.003\,\mathrm{GeV}^2$, and they may be very narrow. In this article, we calculate the values of the strong coupling constants $g_{B_{s0}B_s\eta}$ and $g_{B_{s1}B_s^*\eta}$ with the light-cone QCD sum rules, and study the strong isospin violation decays $B_{s0} \to B_s\pi^0$ and $B_{s1} \to B_s^*\pi^0$. In [31], the authors calculate the strong coupling constants $g_{D_{s0}D_s\eta}$ and $g_{D_{s1}D_s^*\eta}$ with the light-cone QCD sum rules, then take into account η–π^0 mixing and calculate their pionic decay widths.

In the light-cone QCD sum rules approach one carries out the operator product expansion near the light-cone $x^2 \approx 0$ instead of the short distance $x \approx 0$, while the non-perturbative matrix elements are parameterized by the light-cone distribution amplitudes (which are classified according to their twists) instead of vacuum condensates [32–37]. The non-perturbative parameters in the light-cone distribution amplitudes are calculated with the conventional QCD sum rules and the values are universal [38–40].

The article is arranged as follows: in Sect. 2, we derive the strong coupling constants $g_{B_{s0}B_s\eta}$ and $g_{B_{s1}B_s^*\eta}$ with the light-cone QCD sum rules; in Sect. 3, we present the numerical result and a discussion; Sect. 4 is reserved for our conclusions.

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2 Strong coupling constants $g_{B_{s1}B_s^*\eta}$ and $g_{B_{s0}B_s\eta}$ with light-cone QCD sum rules

In the following, we write down the definitions for the strong coupling constants $g_{B_{s0}B_s\eta}$ and $g_{B_{s1}B_s^*\eta}$, respectively:

$$\begin{aligned}\langle B_{s1}|B_s^*\eta\rangle &= -\mathrm{i}g_{B_{s1}B_s^*\eta}\eta^*\cdot\epsilon\,,\\ \langle B_{s0}|B_s\eta\rangle &= g_{B_{s0}B_s\eta}\,,\end{aligned} \tag{1}$$

where ϵ_μ and η_μ are the polarization vectors of the mesons B_s^* and B_{s1}, respectively. The interactions among the bottomed $(0^-,1^-)$, $(0^+,1^+)$ mesons and the light pseudoscalar mesons can be described by the phenomenological lagrangian [41]

$$\begin{aligned}\mathcal{L} &= \mathrm{i}h\mathrm{Tr}\left[S_b\gamma_\mu\gamma_5\mathcal{A}^\mu_{ba}\bar{H}_a\right]+\mathrm{h.c.}\,,\\ S_a &= \frac{1+\not{v}}{2}\left[B^\mu_{a1}\gamma_\mu\gamma_5-B_{a0}\right]\,,\\ H_a &= \frac{1+\not{v}}{2}\left[B^{*\mu}_a\gamma_\mu-\mathrm{i}\gamma_5B_a\right]\,,\\ \bar{H}_a &= \gamma^0H_a^\dagger\gamma^0\,,\\ \mathcal{A}_\mu &= \frac{1}{2}\left(L^\dagger\partial_\mu L-R^\dagger\partial\mu R\right)\,,\\ L &= R^\dagger=\exp\left[\frac{\mathrm{i}\mathcal{M}}{f_\pi}\right]\,,\\ \mathcal{M} &= \begin{pmatrix}\frac{\pi^0}{\sqrt{2}}+\frac{\eta}{\sqrt{6}} & \pi^+ & K^+\\ \pi^- & -\frac{\pi^0}{\sqrt{2}}+\frac{\eta}{\sqrt{6}} & K^0\\ K^- & \bar{K}^0 & -\sqrt{\frac{2}{3}}\eta\end{pmatrix}\,,\end{aligned} \tag{2}$$

where a and b are the flavor indices for the light quarks, $v^2=1$, and h is the strong coupling constant. From the phenomenological lagrangian, we can obtain $g_{B_{s1}B_s^*\eta}\propto \mathrm{i}h$ and $g_{B_{s0}B_s\eta}\propto h$. The hadronic matrix elements $\langle B_{s1}|B_s^*\eta\rangle$ and $\langle B_{s0}|B_s\eta\rangle$ have a relative phase factor i; furthermore, we take the definition $\langle B_{s1}|B_s^*\eta\rangle=-\mathrm{i}g_{B_{s1}B_s^*\eta}\eta^*\cdot\epsilon$ as the one corresponding to $\langle B_{s1}|B^*K\rangle=-\mathrm{i}g_{B_{s1}B^*K}\eta^*\cdot\epsilon$ in [23], where a negative sign is chosen to guarantee that the strong coupling constant $g_{B_{s1}B^*K}$ has a positive value. The expressions in (1) are the correct formula, although there are other definitions [31]. In the literature, the super-fields H_a are usually defined as $H_a=\frac{1+\not{v}}{2}\left[B^{*\mu}_a\gamma_\mu-\gamma_5B_a\right]$; the i that accompany the pseudoscalar mesons B_a is missing; therefore, the i in (1) disappears. Here we take the correct expression given by Manohar and Wise [42].

We study the strong coupling constants $g_{B_{s1}B_s^*\eta}$ and $g_{B_{s0}B_s\eta}$ with the two-point correlation functions $\Pi_{\mu\nu}(p,q)$ and $\Pi_\mu(p,q)$, respectively,

$$\Pi_{\mu\nu}(p,q)=\mathrm{i}\int\mathrm{d}^4x\mathrm{e}^{-\mathrm{i}q\cdot x}\langle 0|T\left\{J_\mu^{\mathrm{V}}(0)J_\nu^{A\dagger}(x)\right\}|\eta(p)\rangle\,, \tag{3}$$

$$\Pi_{\mu}(p,q)=\mathrm{i}\int\mathrm{d}^4x\mathrm{e}^{-\mathrm{i}q\cdot x}\langle 0|T\left\{J_\mu^{5}(0)J^{S\dagger}(x)\right\}|\eta(p)\rangle\,, \tag{4}$$

$$\begin{aligned}J_\mu^{\mathrm{V}}(x) &= \bar{s}(x)\gamma_\mu b(x)\,,\\ J_\mu^{\mathrm{A}}(x) &= \bar{s}(x)\gamma_\mu\gamma_5 b(x)\,,\\ J_\mu^{5}(x) &= \bar{s}(x)\gamma_\mu\gamma_5 b(x)\,,\\ J^{\mathrm{S}}(x) &= \bar{s}(x)b(x)\,,\end{aligned} \tag{5}$$

where the currents $J_\mu^{\mathrm{V}}(x)$, $J_\mu^{\mathrm{A}}(x)$, $J_\mu^5(x)$ and $J^{\mathrm{S}}(x)$ interpolate the strange-bottomed mesons B_s^*, B_{s1}, B_s and B_{s0}, respectively, and the external η meson has four momentum p_μ with $p^2=m_\eta^2$. The $J_\mu^5(x)$ and $J_\mu^{\mathrm{A}}(x)$ are the same current, but we take different notations to express that the contributions are taken from the pseudoscalar meson and axial-vector meson, respectively.

The correlation functions $\Pi_{\mu\nu}(p,q)$ and $\Pi_\mu(p,q)$ can be decomposed as

$$\begin{aligned}\Pi_{\mu\nu}(p,q) &= \mathrm{i}\Pi_{\mathrm{A}}(p,q)g_{\mu\nu}+\mathrm{i}\Pi_{A1}(p,q)p_\mu q_\nu\\ &\quad+\mathrm{i}\Pi_{A2}(p,q)p_\nu q_\mu+\mathrm{i}\Pi_{A3}(p,q)q_\mu q_\nu\,,\\ \Pi_\mu(p,q) &= \mathrm{i}\Pi_{\mathrm{S}}(p,q)q_\mu+\mathrm{i}\Pi_{S1}(p,q)p_\mu\,,\end{aligned} \tag{6}$$

due to the Lorentz invariance. We choose the tensor structures $g_{\mu\nu}$ and q_μ for analysis in this article.

According to the basic assumption of current–hadron duality in the QCD sum rule approach [38–40], we can insert a complete series of intermediate states with the same quantum numbers as the current operators $J_\mu^{\mathrm{V}}(x)$, $J_\mu^{\mathrm{A}}(x)$, $J_\mu^5(x)$ and $J^{\mathrm{S}}(x)$ into the correlation functions $\Pi_{\mu\nu}(p,q)$ and $\Pi_\mu(p,q)$ to obtain the hadronic representations. After isolating the ground state contributions from the pole terms of the mesons B_s^*, B_{s1}, B_s and B_{s0}, we get the following results:

$$\begin{aligned}\Pi_{\mu\nu} &= \frac{\langle 0|J_\mu^{\mathrm{V}}(0)\mid B_s^*(q+p)\rangle\langle B_s^*|B_{s1}\eta\rangle\langle B_{s1}(q)|J_\nu^{\mathrm{A}\dagger}(0)|0\rangle}{\left[M_{B_s^*}^2-(q+p)^2\right]\left[M_{B_{s1}}^2-q^2\right]}\\ &\quad+\frac{\langle 0|J_\mu^{\mathrm{V}}(0)\mid B_s^*(q+p)\rangle\langle B_s^*|B_s\eta\rangle\langle B_s(q)|J_\nu^{\mathrm{A}\dagger}(0)|0\rangle}{\left[M_{B_s^*}^2-(q+p)^2\right]\left[M_{B_s}^2-q^2\right]}\\ &\quad+\frac{\langle 0|J_\mu^{\mathrm{V}}(0)\mid B_{s0}(q+p)\rangle\langle B_{s0}|B_s\eta\rangle\langle B_s(q)|J_\nu^{\mathrm{A}\dagger}(0)|0\rangle}{\left[M_{B_{s0}}^2-(q+p)^2\right]\left[M_{B_s}^2-q^2\right]}\\ &\quad+\cdots\,,\\ &= -\frac{\mathrm{i}g_{B_{s1}B_s^*\eta}f_{B_s^*}f_{B_{s1}}M_{B_s^*}M_{B_{s1}}}{\left[M_{B_s^*}^2-(q+p)^2\right]\left[M_{B_{s1}}^2-q^2\right]}\\ &\quad\times\left[-g_{\mu\lambda}+\frac{(p+q)_\mu(p+q)_\lambda}{M_{B_s^*}^2}\right]\left[-g_{\lambda\nu}+\frac{q_\lambda q_\nu}{M_{B_{s1}}^2}\right]\\ &\quad+\mathrm{i}C_1\left[-g_{\mu\lambda}+\frac{(p+q)_\mu(p+q)_\lambda}{M_{B_s^*}^2}\right]p^\lambda q_\nu\\ &\quad+\mathrm{i}C_2(p+q)_\mu q_\nu+\cdots\,,\\ &= -\frac{\mathrm{i}g_{B_{s1}B_s^*\eta}f_{B_s^*}f_{B_{s1}}M_{B_s^*}M_{B_{s1}}}{\left[M_{B_s^*}^2-(q+p)^2\right]\left[M_{B_{s1}}^2-q^2\right]}g_{\mu\nu}+\cdots\,,\end{aligned} \tag{7}$$

$$\begin{aligned}
\Pi_\mu &= \frac{\langle 0|J^5_\mu(0) \mid B_s(q+p)\rangle\langle B_s|B_{s0}\eta\rangle\langle B_{s0}(q)|J^{S\dagger}(0)|0\rangle}{\left[M^2_{B_s}-(q+p)^2\right]\left[M^2_{B_{s0}}-q^2\right]} \\
&+\frac{\langle 0|J^5_\mu(0) \mid B_{s1}(q+p)\rangle\langle B_{s1}|B_{s0}\eta\rangle\langle B_{s0}(q)|J^{S\dagger}(0)|0\rangle}{\left[M^2_{B_{s1}}-(q+p)^2\right]\left[M^2_{B_{s0}}-q^2\right]} \\
&+\cdots, \\
&= \frac{\mathrm{i}g_{B_{s0}B_s\eta}f_{B_s}f_{B_{s0}}M_{B_{s0}}}{\left[M^2_{B_s}-(q+p)^2\right]\left[M^2_{B_{s0}}-q^2\right]}(p+q)_\mu \\
&+\mathrm{i}C_3\left[-g_{\mu\lambda}+\frac{(p+q)_\mu(p+q)_\lambda}{M^2_{B_{s1}}}\right]p_\lambda+\cdots, \\
&= \frac{\mathrm{i}g_{B_{s0}B_s\eta}f_{B_s}f_{B_{s0}}M_{B_{s0}}}{\left[M^2_{B_s}-(q+p)^2\right]\left[M^2_{B_{s0}}-q^2\right]}q_\mu \\
&+\mathrm{i}C_3\frac{M^2_{B_s^*}+m^2_\eta-M^2_{B_{s1}}}{2M^2_{B_{s1}}}q_\mu+\cdots,
\end{aligned} \tag{8}$$

where the following definitions for the weak decay constants have been used:

$$\begin{aligned}
\langle 0|J^{\mathrm{V}}_\mu(0)|B_s^*(p)\rangle &= f_{B_s^*}M_{B_s^*}\epsilon_\mu, \\
\langle 0|J^{\mathrm{A}}_\mu(0)|B_{s1}(p)\rangle &= f_{B_{s1}}M_{B_{s1}}\eta_\mu, \\
\langle 0|J^5_\mu(0)|B_s(p)\rangle &= \mathrm{i}f_{B_s}p_\mu, \\
\langle 0|J^{\mathrm{S}}(0)|B_{s0}(p)\rangle &= f_{B_{s0}}M_{B_{s0}}, \\
\langle 0|J^{\mathrm{V}}_\mu(0)|B_{s0}(p)\rangle &= f_{B_{s0}}p_\mu.
\end{aligned} \tag{9}$$

We introduce the notation C_i for simplicity; the explicit expressions are ignored as the contributions can be deleted with suitable tensor structures. The term proportional to the C_3 is greatly suppressed by the small numerical factor $\frac{M^2_{B_s^*}+m^2_\eta-M^2_{B_{s1}}}{M^2_{B_{s1}}}$, and the contributions from the axial-vector meson can be neglected safely in (8). We choose the tensor structure $g_{\mu\nu}$ to avoid the contaminations from the scalar meson B_{s0} and the pseudoscalar meson B_s in the sum rule for the strong coupling constant $g_{B_{s1}B_s^*\eta}$. In deriving the sum rule for the strong coupling constant $g_{B_{s0}B_s\eta}$, we choose the axial-vector current $J^5_\mu(x)$ to interpolate the pseudoscalar meson B_s, although there are contaminations from the axial-vector meson B_{s1}; the contaminations are tiny and can be ignored safely if we choose the tensor structure q_μ. If we choose the pseudoscalar current $J_5(x)=\bar{s}(x)\mathrm{i}\gamma_5 c(x)$ to interpolate the pseudoscalar meson B_s, the axial-vector mesons have no contaminations; I failed to take notice of this fact at the beginning of the work.

We perform the operator product expansion for the correlation functions $\Pi_{\mu\nu}(p,q)$ and $\Pi_\mu(p,q)$ in perturbative QCD theory and obtain the analytical expressions at the level of the quark-gluon degrees of freedom. In the calculation, the two-particle and three-particle η meson light-cone distribution amplitudes have been used [43, 44], and the explicit expressions are given in the appendix. The parameters in the light-cone distribution amplitudes are scale dependent and are calculated with the QCD sum rules [43, 44]. In this article, the energy scale μ is chosen to be $\mu = 1$ GeV, but one can choose another typical energy scale: $\mu=\sqrt{M_B^2-m_b^2}\approx 2.4$ GeV. The light-cone distribution amplitudes are calculated at the energy scale $\mu = 1$ GeV with the QCD sum rules; on evolution of the coefficients to larger energy scales with the (complex) renormalization group equation, which concerns approximations in one way or the other, additional uncertainties are introduced. The physical quantities would not depend on the special energy scale we choose, and we expect that the scale dependence of the input parameters is approximately canceled, so the values of the strong coupling constants which are calculated at the energy scale $\mu = 1$ GeV can make robust predictions. Furthermore, in the heavy quark limit, the bound energy of the strange-bottomed $(0^+,1^+)$ mesons is about $\Lambda=\frac{3M_{B_{s1}}+M_{B_{s0}}}{4}-m_b\approx 1$ GeV, which can serve as a typical energy scale and validate our choice.

After straightforward calculations, we obtain the final expressions of the double Borel transformed correlation functions Π_{A} and Π_{S} at the level of quark-gluon degrees of freedom. The masses of the strange-bottomed mesons are $M_{B_{s1}}=5.72$ GeV, $M_{B_{s0}}=5.70$ GeV, $M_{B_s^*}=5.412$ GeV and $M_{B_s}=5.366$ GeV,

$$\frac{M^2_{B_{s1}}}{M^2_{B_{s1}}+M^2_{B_s^*}}\approx\frac{M^2_{B_{s0}}}{M^2_{B_{s0}}+M^2_{B_s}}\approx 0.53\,, \tag{10}$$

and there exists an overlapping working window for the two Borel parameters M_1^2 and M_2^2. It is convenient to take the value $M_1^2=M_2^2$. We introduce the threshold parameter s_0 (this denotes s^0_{S} and s^0_{A}) and make the simple replacement

$$\mathrm{e}^{-\frac{m_b^2+u_0(1-u_0)m_\eta^2}{M^2}}\to\mathrm{e}^{-\frac{m_b^2+u_0(1-u_0)m_\eta^2}{M^2}}-\mathrm{e}^{-\frac{s_0}{M^2}}$$

to subtract the contributions from the high resonances and continuum states [45]; finally, we obtain the sum rules for the strong coupling constants $g_{B_{s0}B_s\eta}$ and $g_{B_{s1}B_s^*\eta}$, respectively,[1]

$$\begin{aligned}
g_{B_{s0}B_s\eta} &= \frac{1}{f_{B_s}f_{B_{s0}}M_{B_{s0}}}\exp\left(\frac{M^2_{B_{s0}}}{M_1^2}+\frac{M^2_{B_s}}{M_2^2}\right) \\
&\times\left\{\left[\exp\left(-\frac{\Xi}{M^2}\right)-\exp\left(-\frac{s^0_{\mathrm{S}}}{M^2}\right)\right]\right. \\
&\times\frac{f'_\eta m_\eta^2M^2}{m_s}\left[\varphi_p(u_0)-\frac{\mathrm{d}\varphi_\sigma(u_0)}{6\,\mathrm{d}u_0}\right] \\
&+\exp\left(-\frac{\Xi}{M^2}\right)\left[-m_bf'_\eta m_\eta^2\int_0^{u_0}\mathrm{d}tB(t)\right. \\
&+f'_{3\eta}m_\eta^2\int_0^{u_0}\mathrm{d}\alpha_s \\
&\times\int_{u_0-\alpha_s}^{1-\alpha_s}d\alpha_g\varphi_{3\eta}(1-\alpha_s-\alpha_g,\alpha_g,\alpha_s)
\end{aligned}$$

[1] For example, we use the notation $(A_\parallel+A_\perp)(1-\alpha-\beta,\alpha,\beta)$ to represent $A_\parallel(1-\alpha-\beta,\alpha,\beta)+A_\perp(1-\alpha-\beta,\alpha,\beta)$. Other expressions can be understood in the same way.

$$
\begin{aligned}
&\times \frac{2(\alpha_s+\alpha_g-u_0)-3\alpha_g}{\alpha_g^2} \\
&-\frac{2m_b f'_\eta m_\eta^4}{M^2}\int_{1-u_0}^{1} \mathrm{d}\alpha_g \frac{1-u_0}{\alpha_g^2}\int_0^{\alpha_g} \mathrm{d}\beta \\
&\times \int_0^{1-\beta} \mathrm{d}\alpha \Phi(1-\alpha-\beta,\beta,\alpha) \\
&+\frac{2m_b f'_\eta m_\eta^4}{M^2}\left(\int_0^{1-u_0}\mathrm{d}\alpha_g\int_{u_0-\alpha_g}^{u_0}\mathrm{d}\alpha_s\int_0^{\alpha_s}\mathrm{d}\alpha \right.\\
&\left.+\int_{1-u_0}^{1}\mathrm{d}\alpha_g\int_{u_0-\alpha_g}^{1-\alpha_g}\mathrm{d}\alpha_s\int_0^{\alpha_s}\mathrm{d}\alpha\right)\\
&\left.\left.\times \frac{\Phi(1-\alpha-\alpha_g,\alpha_g,\alpha)}{\alpha_g}\right]\right\}, \qquad (11)
\end{aligned}
$$

$$
\begin{aligned}
g_{B_{s1}B_s^*\eta} =& \frac{1}{f_{B_s^*}f_{B_{s1}}M_{B_s^*}M_{B_{s1}}}\exp\left(\frac{M_{B_{s1}}^2}{M_1^2}+\frac{M_{B_s^*}^2}{M_2^2}\right)\\
&\times\left\{\left[\exp\left(-\frac{\Xi}{M^2}\right)-\exp\left(-\frac{s_A^0}{M^2}\right)\right]\right.\\
&\times f'_\eta\left[\frac{m_b m_\eta^2 M^2}{m_s}\varphi_p(u_0)\right.\\
&\left.+\frac{m_\eta^2(M^2+m_b^2)}{8}\frac{\mathrm{d}}{\mathrm{d}u_0}A(u_0)-\frac{M^4}{2}\frac{\mathrm{d}}{\mathrm{d}u_0}\phi_\eta(u_0)\right]\\
&-\exp\left(-\frac{\Xi}{M^2}\right)\left[f'_\eta m_b^2 m_\eta^2\int_0^{u_0}\mathrm{d}tB(t)\right.\\
&+m_\eta^2\int_0^{u_0}\mathrm{d}\alpha_s\int_{u_0-\alpha_s}^{1-\alpha_s}\mathrm{d}\alpha_g\\
&\times\frac{(u_0 f'_\eta m_\eta^2\Phi+f'_{3\eta}m_b\varphi_{3\eta})(1-\alpha_s-\alpha_g,\alpha_s,\alpha_g)}{\alpha_g}\\
&+f'_\eta m_\eta^2 M^2\frac{\mathrm{d}}{\mathrm{d}u_0}\int_0^{u_0}\mathrm{d}\alpha_s\int_{u_0-\alpha_s}^{1-\alpha_s}\mathrm{d}\alpha_g\\
&\times\frac{(A_\parallel-V_\parallel)(1-\alpha_s-\alpha_g,\alpha_s,\alpha_g)}{2\alpha_g}\\
&-f'_\eta m_\eta^2 M^2\frac{\mathrm{d}}{\mathrm{d}u_0}\int_0^{u_0}\mathrm{d}\alpha_s\int_{u_0-\alpha_s}^{1-\alpha_s}\mathrm{d}\alpha_g\\
&\times A_\parallel(1-\alpha_s-\alpha_g,\alpha_s,\alpha_g)\frac{\alpha_s+\alpha_g-u_0}{\alpha_g^2}\\
&+f'_\eta m_\eta^4\left(\int_0^{1-u_0}\mathrm{d}\alpha_g\int_{u_0-\alpha_g}^{u_0}\mathrm{d}\alpha_s\int_0^{\alpha_s}\mathrm{d}\alpha\right.\\
&\left.+\int_{1-u_0}^{1}\mathrm{d}\alpha_g\int_{u_0-\alpha_g}^{1-\alpha_g}\mathrm{d}\alpha_s\int_0^{\alpha_s}\mathrm{d}\alpha\right)\\
&\times\left[\frac{1}{\alpha_g}\left(3-\frac{2m_b^2}{M^2}\right)\Phi\right.\\
&\left.+\frac{4m_b^2}{M^2}\frac{\alpha_s+\alpha_g-u_0}{\alpha_g^2}(A_\perp+A_\parallel)\right]\\
&\times(1-\alpha-\alpha_g,\alpha,\alpha_g)\\
&-f'_\eta m_\eta^4 u_0\frac{\mathrm{d}}{\mathrm{d}u_0}\left(\int_0^{1-u_0}\mathrm{d}\alpha_g\int_{u_0-\alpha_g}^{u_0}\mathrm{d}\alpha_s\int_0^{\alpha_s}\mathrm{d}\alpha\right.\\
&\left.+\int_{1-u_0}^{1}\mathrm{d}\alpha_g\int_{u_0-\alpha_g}^{1-\alpha_g}\mathrm{d}\alpha_s\int_0^{\alpha_s}\mathrm{d}\alpha\right)\\
&\times\frac{\Phi(1-\alpha-\alpha_g,\alpha,\alpha_g)}{\alpha_g}\\
&-f'_\eta m_\eta^4\int_{1-u_0}^{1}\mathrm{d}\alpha_g\int_0^{\alpha_g}\mathrm{d}\beta\int_0^{1-\beta}\mathrm{d}\alpha\\
&\times\left[\Phi(1-\alpha-\beta,\alpha,\beta)\frac{1-u_0}{\alpha_g^2}\left(4-\frac{2m_b^2}{M^2}\right)\right.\\
&\left.+\frac{4m_b^2}{M^2}\frac{(1-u_0)^2}{\alpha_g^3}(A_\parallel+A_\perp)(1-\alpha-\beta,\alpha,\beta)\right]\\
&+f'_\eta m_\eta^4\frac{\mathrm{d}}{\mathrm{d}u_0}\int_{1-u_0}^{1}\mathrm{d}\alpha_g\int_0^{\alpha_g}\mathrm{d}\beta\int_0^{1-\beta}\mathrm{d}\alpha\\
&\left.\left.\times\Phi(1-\alpha-\beta,\alpha,\beta)\frac{u_0(1-u_0)}{\alpha_g^2}\right]\right\}, \qquad (12)
\end{aligned}
$$

where

$$
\begin{aligned}
\Phi(\alpha_i) &= A_\parallel(\alpha_i)+A_\perp(\alpha_i)-V_\parallel(\alpha_i)-V_\perp(\alpha_i)\,,\\
\Xi &= m_b^2+u_0(1-u_0)m_\eta^2\,,\\
u_0 &= \frac{M_1^2}{M_1^2+M_2^2}\,,\\
M^2 &= \frac{M_1^2M_2^2}{M_1^2+M_2^2}\,. \qquad (13)
\end{aligned}
$$

3 Numerical result and discussion

The input parameters are taken as $m_s = (140\pm10)$ MeV, $m_b = (4.7\pm0.1)$ GeV, $\lambda_3 = 0.0$, $a_1 = 0.0$, $f_{3\eta} = (0.40\pm0.12)\times10^{-2}\,\mathrm{GeV}^2$, $\omega_3 = -3.0\pm0.9$, $\eta_4 = 0.5\pm0.2$, $\omega_4 = 0.2\pm0.1$, $a_2 = 0.20\pm0.06$ [43, 44], $f_\eta = 0.145$ GeV, $m_\eta = 0.548$ GeV, $f'_\eta = -\frac{2}{\sqrt{6}}f_\eta$, $f'_{3\eta} = -\frac{2}{\sqrt{6}}f_{3\eta}$, $M_{B_s} = 5.366$ GeV, $M_{B_s^*} = 5.412$ GeV [46], $M_{B_{s0}} = (5.70\pm0.11)$ GeV, $M_{B_{s1}} = (5.72\pm0.09)$ GeV, $f_{B_{s0}} = f_{B_{s1}} = (0.24\pm0.02)$ GeV [17], $f_{B_s^*} = f_{B_s} = (0.19\pm0.02)$ GeV [37, 47, 48], $s_S^0 = (37\pm1)\,\mathrm{GeV}^2$ and $s_A^0 = (38\pm1)\,\mathrm{GeV}^2$ [17]. The Borel parameters are chosen as $M^2 = (5-7)\,\mathrm{GeV}^2$, and in this region, the values of the strong coupling constants $g_{B_{s1}B_s^*\eta}$ and $g_{B_{s0}B_s\eta}$ are rather stable, as shown in Fig. 1.

In the limit of large Borel parameter M^2, the strong coupling constants $g_{B_{s1}B_s^*\eta}$ and $g_{B_{s0}B_s\eta}$ take up the following behaviors, respectively:

$$
\begin{aligned}
g_{B_{s0}B_s\eta} &\propto \frac{M^2\varphi_p(u_0)}{f_{B_s}f_{B_{s0}}}\,,\\
g_{B_{s1}B_s^*\eta} &\propto \frac{m_b M^2\varphi_p(u_0)}{f_{B_s^*}f_{B_{s1}}}\,. \qquad (14)
\end{aligned}
$$

It is not unexpected that the contributions from the two-particle twist-3 light-cone distribution amplitude $\varphi_p(u)$ are greatly enhanced by the large Borel parameter M^2; (large) uncertainties of the relevant parameters present in the

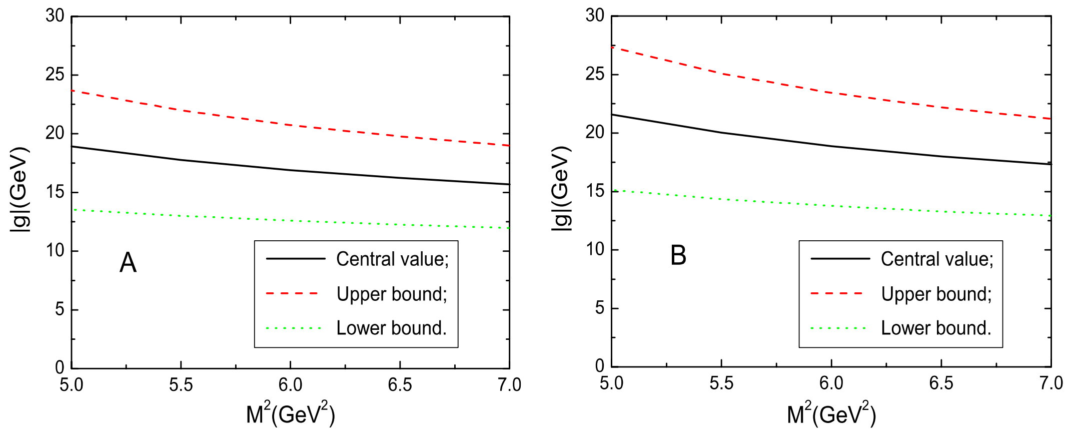

Fig. 1. The strong coupling constants $g_{B_{s1}B_s^*\eta}$ (**a**) and $g_{B_{s0}B_s\eta}$ (**b**) with the parameter M^2

above equations have a significant impact on the numerical results. The contribution from the two-particle twist-3 light-cone distribution amplitude $\varphi_\sigma(u_0)$ is zero due to symmetry.

Taking into account all the uncertainties of the input parameters, finally we obtain the numerical values of the strong coupling constants, which are shown in Fig. 1,

$$\begin{aligned}|g_{B_{s1}B_s^*\eta}| &= (17.8\pm5.8)\,\mathrm{GeV}\,,\\ |g_{B_{s0}B_s\eta}| &= (20.1\pm7.2)\,\mathrm{GeV}\,,\end{aligned} \tag{15}$$

and the uncertainties are large, about 30%. Taking into account the small η–π^0 transition matrix according to Dashen's theorem [30], $t_{\eta\pi} = \langle\pi^0|\mathcal{H}|\eta\rangle = -0.003\,\mathrm{GeV}^2$, we can obtain the narrow decay widths

$$\begin{aligned}\Gamma_{B_{s1}B_s^*\pi} &= \frac{p_1}{24\pi M_{B_{s1}}^2}\sum_\lambda\sum_{\lambda'}\left|\frac{g_{B_{s1}B_s^*\eta}\eta^*(\lambda)\cdot\epsilon(\lambda')t_{\eta\pi}}{m_\pi^2-m_\eta^2}\right|^2\\ &= (5.3\text{–}20.7)\,\mathrm{keV}\,,\\ \Gamma_{B_{s0}B_s\pi} &= \frac{p_2}{8\pi M_{B_{s0}}^2}\left|\frac{g_{B_{s0}B_s\eta}t_{\eta\pi}}{m_\pi^2-m_\eta^2}\right|^2 = (6.8\text{–}30.7)\,\mathrm{keV}\,,\end{aligned} \tag{16}$$

$$p_1 = \frac{\sqrt{\left[M_{B_{s1}}^2-(M_{B_s^*}+m_\pi)^2\right]\left[M_{B_{s1}}^2-(M_{B_s^*}-m_\pi)^2\right]}}{2M_{B_{s1}}}\,,$$
$$p_2 = \frac{\sqrt{\left[M_{B_{s0}}^2-(M_{B_s}+m_\pi)^2\right]\left[M_{B_{s0}}^2-(M_{B_s}-m_\pi)^2\right]}}{2M_{B_{s0}}}\,,$$

which are consistent with the ones obtained from the analysis of the unitarized two-meson scattering amplitudes with the heavy-light chiral lagrangian, $\Gamma_{B_{s1}B_s^*\pi} = 10.36\,\mathrm{keV}$ and $\Gamma_{B_{s0}B_s\pi} = 7.92\,\mathrm{keV}$ [49, 50]. We can search the strange-bottomed $(0^+,1^+)$ mesons B_{s0} and B_{s1} in the invariant $B_s\pi^0$ and $B_s^*\pi^0$ mass distributions respectively, just like the BaBar and CLEO Collaborations observed the strange-charmed $(0^+,1^+)$ mesons D_{s0} and D_{s1} in the invariant $D_s\pi^0$ and $D_s^*\pi^0$ mass distributions, respectively [51, 52].

4 Conclusion

In this article, we calculate the strong coupling constants $g_{B_{s0}B_s\eta}$ and $g_{B_{s1}B_s^*\eta}$ with the light-cone QCD sum rules. Then we take into account the small η–π^0 transition matrix according to Dashen's theorem and obtain the small decay widths. We can search the strange-bottomed $(0^+,1^+)$ mesons B_{s0} and B_{s1} in the invariant $B_s\pi^0$ and $B_s^*\pi^0$ mass distributions, respectively.

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Appendix

The light-cone distribution amplitudes of the η meson are defined by

$$\begin{aligned}&\langle 0|\bar{s}(0)\gamma_\mu\gamma_5 s(x)|\eta(p)\rangle\\ &= \mathrm{i}f'_\eta p_\mu\int_0^1 \mathrm{d}u\,\mathrm{e}^{-\mathrm{i}up\cdot x}\left\{\phi_\eta(u)+\frac{m_\eta^2x^2}{16}A(u)\right\}\\ &\quad + f'_\eta m_\eta^2\frac{\mathrm{i}x_\mu}{2p\cdot x}\int_0^1 \mathrm{d}u\,\mathrm{e}^{-\mathrm{i}up\cdot x}B(u)\,,\\ &\langle 0|\bar{s}(0)\mathrm{i}\gamma_5 s(x)|\eta(p)\rangle = \frac{f'_\eta m_\eta^2}{m_s}\int_0^1 \mathrm{d}u\,\mathrm{e}^{-\mathrm{i}up\cdot x}\varphi_p(u)\,,\\ &\langle 0|\bar{s}(0)\sigma_{\mu\nu}\gamma_5 s(x)|\eta(p)\rangle\\ &= \mathrm{i}(p_\mu x_\nu-p_\nu x_\mu)\frac{f'_\eta m_\eta^2}{6m_s}\int_0^1 \mathrm{d}u\,\mathrm{e}^{-\mathrm{i}up\cdot x}\varphi_\sigma(u)\,,\\ &\langle 0|\bar{s}(0)\sigma_{\alpha\beta}\gamma_5 g_s G_{\mu\nu}(vx)s(x)|\eta(p)\rangle\\ &= f'_{3\eta}\left\{(p_\mu p_\alpha g^\perp_{\nu\beta}-p_\nu p_\alpha g^\perp_{\mu\beta})-(p_\mu p_\beta g^\perp_{\nu\alpha}\right.\\ &\quad\left.-p_\nu p_\beta g^\perp_{\mu\alpha})\right\}\int\mathcal{D}\alpha_i\varphi_{3\eta}(\alpha_i)\,\mathrm{e}^{-\mathrm{i}p\cdot x(\alpha_s+v\alpha_g)}\,,\\ &\langle 0|\bar{s}(0)\gamma_\mu\gamma_5 g_s G_{\alpha\beta}(vx)s(x)|\eta(p)\rangle\\ &= p_\mu\frac{p_\alpha x_\beta-p_\beta x_\alpha}{p\cdot x}f'_\eta m_\eta^2\int\mathcal{D}\alpha_i A_\parallel(\alpha_i)\,\mathrm{e}^{-\mathrm{i}p\cdot x(\alpha_s+v\alpha_g)}\\ &\quad + f'_\eta m_\eta^2(p_\beta g_{\alpha\mu}-p_\alpha g_{\beta\mu})\int\mathcal{D}\alpha_i A_\perp(\alpha_i)\,\mathrm{e}^{-\mathrm{i}p\cdot x(\alpha_s+v\alpha_g)}\,,\end{aligned}$$

$$\begin{aligned}
&\langle 0|\bar{s}(0)\gamma_\mu g_s \tilde{G}_{\alpha\beta}(vx)s(x)|\eta(p)\rangle \\
&= p_\mu \frac{p_\alpha x_\beta - p_\beta x_\alpha}{p\cdot x} f'_\eta m_\eta^2 \int \mathcal{D}\alpha_i V_\parallel(\alpha_i)\,\mathrm{e}^{-\mathrm{i}p\cdot x(\alpha_s+v\alpha_g)} \\
&\quad + f'_\eta m_\eta^2 (p_\beta g_{\alpha\mu} - p_\alpha g_{\beta\mu}) \int \mathcal{D}\alpha_i V_\perp(\alpha_i)\,\mathrm{e}^{-\mathrm{i}p\cdot x(\alpha_s+v\alpha_g)}\,,
\end{aligned} \tag{A.1}$$

where the operator $\tilde{G}_{\alpha\beta}$ is the dual of the $G_{\alpha\beta}$, $\tilde{G}_{\alpha\beta} = \frac{1}{2}\epsilon_{\alpha\beta\mu\nu}G^{\mu\nu}$ and $\mathcal{D}\alpha_i$ is defined as $\mathcal{D}\alpha_i = \mathrm{d}\alpha_{\bar{s}}\mathrm{d}\alpha_g\mathrm{d}\alpha_s\delta(1-\alpha_{\bar{s}}-\alpha_g-\alpha_s)$. The light-cone distribution amplitudes are parameterized as

$$\begin{aligned}
\phi_\eta(u) &= 6u(1-u)\left\{1 + a_1 C_1^{\frac{3}{2}}(2u-1) + a_2 C_2^{\frac{3}{2}}(2u-1)\right\}, \\
\varphi_p(u) &= 1 + \left\{30\eta_3 - \frac{5}{2}\rho^2\right\} C_2^{\frac{1}{2}}(2u-1) \\
&\quad + \left\{-3\eta_3\omega_3 - \frac{27}{20}\rho^2 - \frac{81}{10}\rho^2 a_2\right\} C_4^{\frac{1}{2}}(2u-1)\,, \\
\varphi_\sigma(u) &= 6u(1-u)\left\{1 + \left[5\eta_3 - \frac{1}{2}\eta_3\omega_3 - \frac{7}{20}\rho^2 - \frac{3}{5}\rho^2 a_2\right]\right. \\
&\quad \left.\times C_2^{\frac{3}{2}}(2u-1)\right\}, \\
\varphi_{3\eta}(\alpha_i) &= 360\alpha_{\bar{s}}\alpha_s\alpha_g^2\left\{1 + \lambda_3(\alpha_{\bar{s}}-\alpha_s) + \omega_3\frac{1}{2}(7\alpha_g-3)\right\}, \\
V_\parallel(\alpha_i) &= 120\alpha_{\bar{s}}\alpha_s\alpha_g\left(v_{00} + v_{10}(3\alpha_g-1)\right), \\
A_\parallel(\alpha_i) &= 120\alpha_{\bar{s}}\alpha_s\alpha_g a_{10}(\alpha_s-\alpha_{\bar{s}})\,, \\
V_\perp(\alpha_i) &= -30\alpha_g^2\left\{h_{00}(1-\alpha_g) + h_{01}\left[\alpha_g(1-\alpha_g) - 6\alpha_{\bar{s}}\alpha_s\right]\right. \\
&\quad \left. + h_{10}\left[\alpha_g(1-\alpha_g) - \frac{3}{2}\left(\alpha_{\bar{s}}^2+\alpha_s^2\right)\right]\right\}, \\
A_\perp(\alpha_i) &= 30\alpha_g^2(\alpha_{\bar{s}}-\alpha_s)\left\{h_{00} + h_{01}\alpha_g + \frac{1}{2}h_{10}(5\alpha_g-3)\right\}, \\
A(u) &= 6u(1-u)\left\{\frac{16}{15} + \frac{24}{35}a_2 + 20\eta_3 + \frac{20}{9}\eta_4\right. \\
&\quad + \left[-\frac{1}{15} + \frac{1}{16} - \frac{7}{27}\eta_3\omega_3 - \frac{10}{27}\eta_4\right] C_2^{\frac{3}{2}}(2u-1) \\
&\quad \left. + \left[-\frac{11}{210}a_2 - \frac{4}{135}\eta_3\omega_3\right] C_4^{\frac{3}{2}}(2u-1)\right\} \\
&\quad + \left\{-\frac{18}{5}a_2 + 21\eta_4\omega_4\right\} \\
&\quad \times \left\{2u^3(10-15u+6u^2)\log u\right. \\
&\quad + 2\bar{u}^3(10-15\bar{u}+6\bar{u}^2)\log\bar{u} \\
&\quad \left. + u\bar{u}(2+13u\bar{u})\right\}, \\
g_\eta(u) &= 1 + g_2 C_2^{\frac{1}{2}}(2u-1) + g_4 C_4^{\frac{1}{2}}(2u-1)\,, \\
B(u) &= g_\eta(u) - \phi_\eta(u)\,,
\end{aligned} \tag{A.2}$$

where

$$\begin{aligned}
h_{00} &= v_{00} = -\frac{\eta_4}{3}\,, \\
a_{10} &= \frac{21}{8}\eta_4\omega_4 - \frac{9}{20}a_2\,, \\
v_{10} &= \frac{21}{8}\eta_4\omega_4\,, \\
h_{01} &= \frac{7}{4}\eta_4\omega_4 - \frac{3}{20}a_2\,, \\
h_{10} &= \frac{7}{2}\eta_4\omega_4 + \frac{3}{20}a_2\,, \\
g_2 &= 1 + \frac{18}{7}a_2 + 60\eta_3 + \frac{20}{3}\eta_4\,, \\
g_4 &= -\frac{9}{28}a_2 - 6\eta_3\omega_3\,;
\end{aligned} \tag{A.3}$$

here $C_2^{\frac{1}{2}}(\xi)$, $C_4^{\frac{1}{2}}(\xi)$ and $C_2^{\frac{3}{2}}(\xi)$ are Gegenbauer polynomials, $\eta_3 = \frac{f_{3\eta}}{f_\eta}\frac{m_s}{m_\eta^2}$ and $\rho^2 = \frac{m_s^2}{m_\eta^2}$ [43, 44].

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